A brief introduction to **computational complexity**

Arnaud Casteigts
LaBRI, Université de Bordeaux

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Study of the computational resources required for solving a given **problem**.

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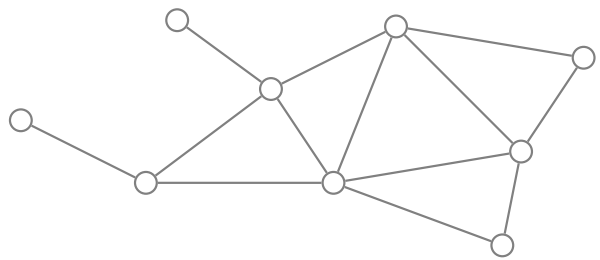

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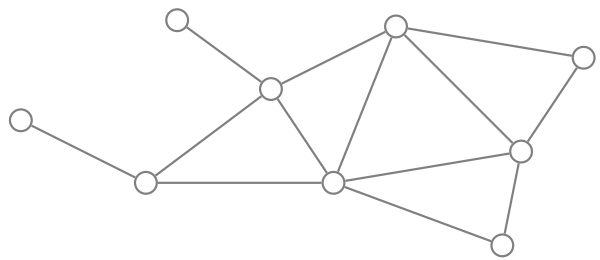

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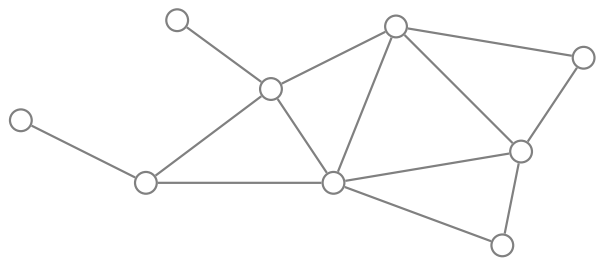

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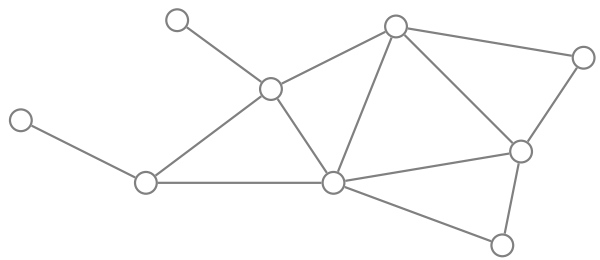

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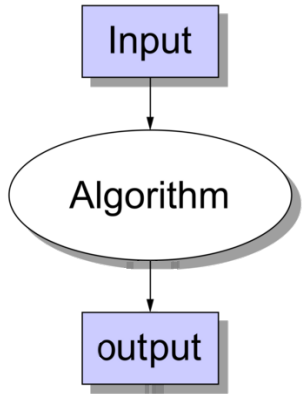

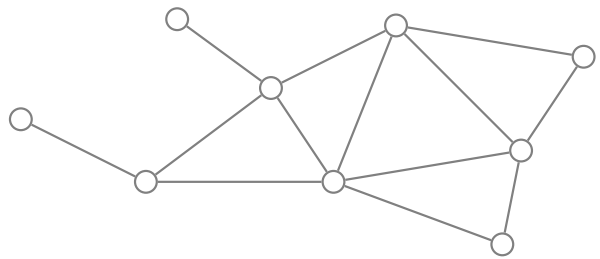

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Types of problems

Different types of problems (examples based on coloring of a graph G)

- **Decision:** Does G admit a k-coloring (fir given k)?
- **Search:** Find a k-coloring of G (for a given k).
- **Counting:** How many k-colorings does G admit (for a given k)?
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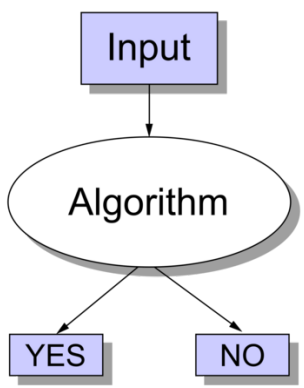
Decision problem (focus)

Special case where $\Pi : \{0,1\}^* \to \{0,1\}$ (the answer is YES or NO)

$x \in \{0,1\}^*$ is a **positive instance** if $\Pi(x) = 1$ (resp. **negative** if 0)

Positive instances define a "**formal language**" (ie a **set of words**) $L \subseteq \{0,1\}^*$

Point of view:
Solving a decision problem goes down to deciding if a given word is in L



What questions?

Goal of computational complexity:

$\rightarrow$ classify the problems according to the resources needed for solving them.

What type of resources?

- Time (= number of operations)
- Space (= memory)
- Randomness (number of random bits)
- Non-determinism
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Asymptotic point of view

- What matters is how these quantities **scale** with the **size** of the input, noted $\boldsymbol{n}$.
- Notations $O(\cdot), \Omega(\cdot), \Theta(\cdot), o(\cdot), \omega(\cdot)$

  Intuition $\leq \quad \geq \quad = \quad < \quad >$ for $n \to \infty$, up to constant factors
- Examples:

Constant	$O(1)$	Quadratic	$O(n^2)$
Logarithmic	$O(\log n)$	Polynomial	$O(n^c) = n^{O(1)} = poly(n)$
Linear	$O(n)$	Exponential	$O(2^n)$ ou $O(2^{poly(n)})$
Quasi-linear	$O(n \log n)$	Factorial	$O(n^n)$

Space and time

Unless otherwise mentioned, let us focus on **decision** problems

Generic classes

- $\text{TIME}(f(n))$: Set of problems solvable in time $O(f(n))$.
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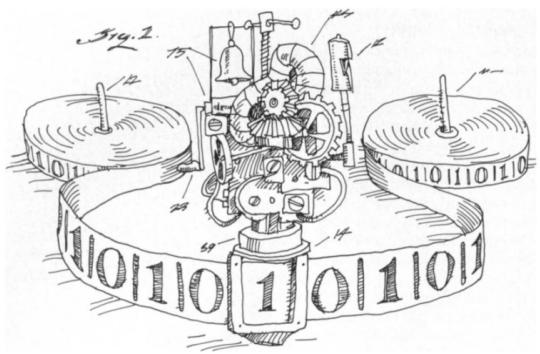
Machine model?

- Turing machines
- Intuition: “standard algorithm” OK
- Universality?



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Theorems and hierarchies (simplified)

Given two fonctions $f_1(n)$ and $f_2(n)$ (abbreviated as f_1 and f_2 below):

- Time hierarchy:
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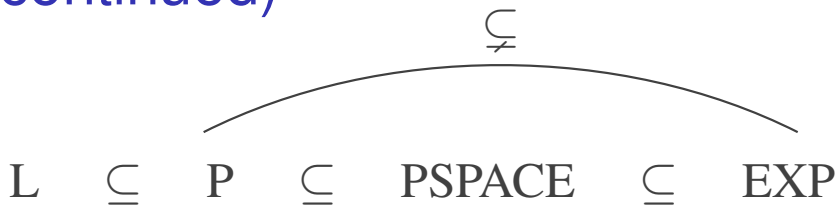

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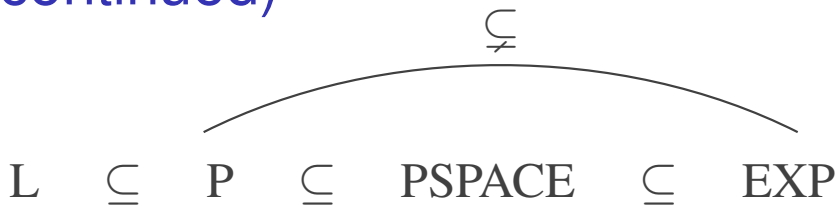

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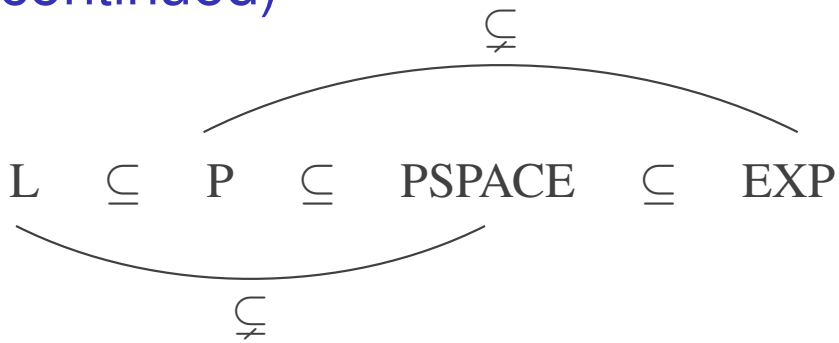

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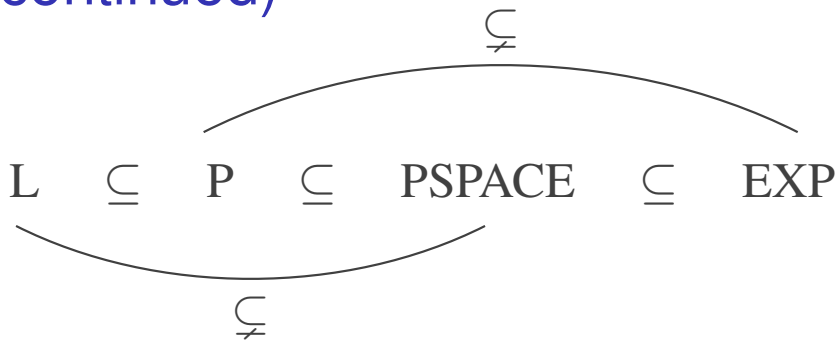

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Space versus time?

- $\text{TIME}(f(n)) \subseteq \text{SPACE}(f(n)/\log(f(n)))$ (Hopcroft, Paul, Valiant'77)
  $\rightarrow$ Space is **strictly** better than time!

Non-determinism (NP and coNP)

Several equivalent definitions, the simplest are:

- NP: $\exists$ **short proof** when the answer is YES (positive certificate)
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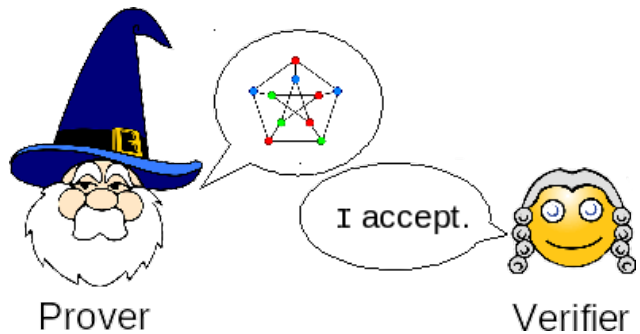

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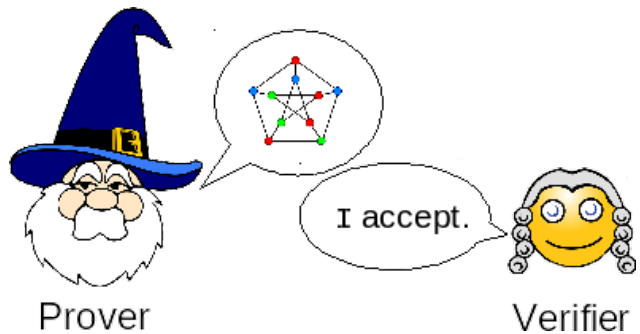
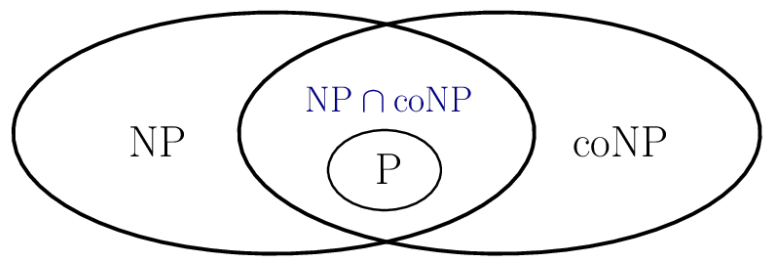


Exercise: categorize the following problems

- Is a given graph connected?
- Is a given number prime? is it composite?
- Does this number admit a factor $\leq k$?
- Are these two graphs isomorphic?
- Does a given graph admit a clique of size $n/2$?
- Does a given $n \times n$ chess board admit a winning strategy for player 1?
- Does this program always terminate?



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More generally

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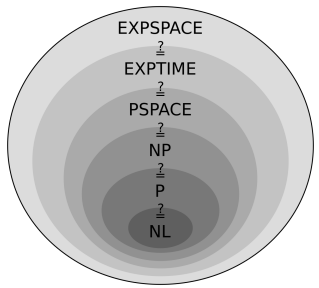

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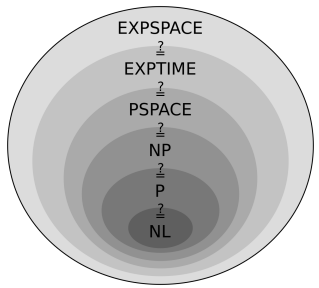

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- Padding arguments: $\mathrm{P} = \mathrm{NP} \implies \mathrm{EXP} = \mathrm{NEXP}$ (et d'autres)

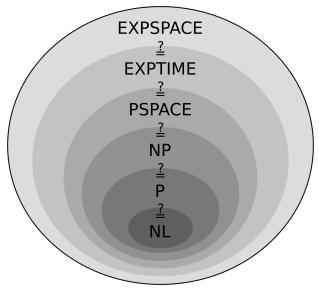
Non-determinism (continued)

More generally

- $\mathrm{NTIME}(f(n))$: $\exists$ positive certificates verifiable in time $O(f(n))$.
- $\mathrm{NSPACE}(f(n))$: $\exists$ positive certificates verifiable in space $O(f(n))$.
- $\mathrm{coNTIME}$ and $\mathrm{coNSPACE}$: same for negative certificates.

Main classes (same as before, but non-deterministic version)

$$
\begin{array}{lcl}
\mathrm{NL} & := & \mathrm{NSPACE}(\log n) \\
\mathrm{NP} & := & \mathrm{NTIME}(poly(n)) \\
\mathrm{NPSPACE} & := & \mathrm{NSPACE}(poly(n)) \\
\mathrm{NEXP} & := & \mathrm{NTIME}(2^{poly(n)})
\end{array}
$$



Relations

- $\mathrm{P} \subseteq \mathrm{NP} \cap \mathrm{coNP}$ (algo = verifier with an empty certificate)
- $\mathrm{NSPACE}(f(n)) \subseteq \mathrm{SPACE}(f^2(n))$ (Savitch'70)
- $\mathrm{NSPACE}(f(n)) = \mathrm{coNSPACE}(f(n))$ (Immerman–Szelepcsényi'87)
- $\mathrm{NTIME}(f(n)) \subseteq \mathrm{SPACE}(f(n))$ (Certificate enumeration)
- $\mathrm{L} \stackrel{?}{=} \mathrm{NL} \stackrel{?}{=} \mathrm{P} \stackrel{?}{=} \mathrm{NP} \stackrel{?}{=} \mathrm{PSPACE} \stackrel{?}{=} \mathrm{EXP} \stackrel{?}{=} \mathrm{NEXP} \stackrel{?}{=} \mathrm{EXPSPACE}$ (Inclusions OK)
- Padding arguments: $\mathrm{P} = \mathrm{NP} \implies \mathrm{EXP} = \mathrm{NEXP}$ (et d'autres)
- $\mathbf{P} \stackrel{?}{=} \mathbf{NP}$ **Does "easy to check" implies "easy to compute"?**

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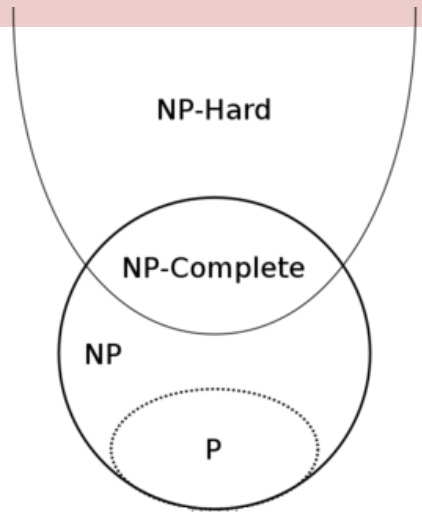
Ex: NP-complete = in NP and NP-hard

(here, *transformation-based* and *in polynomial time*)

SAT $\in$ NP-complet (Cook, Levin’71)



(what people believe)

Thanks!