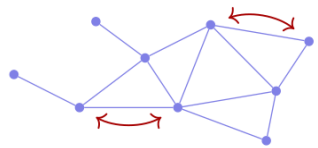
Distributed algorithm

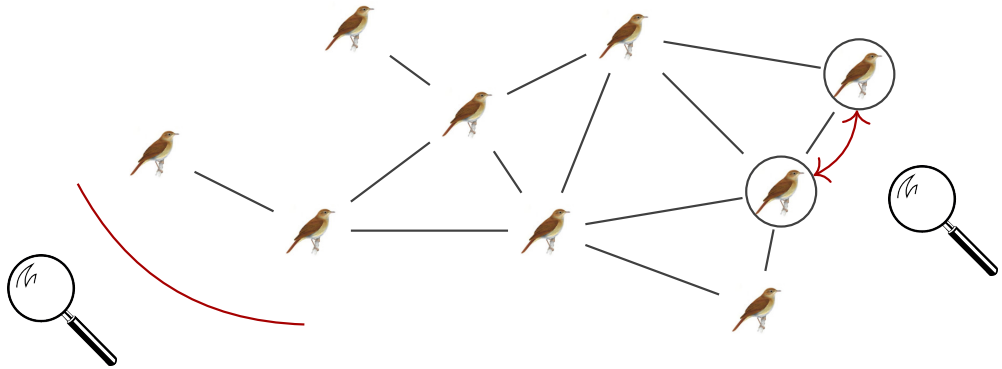
Arnaud Casteigts



Part of Graph Algorithms (14X061)
Masters of Computer Science

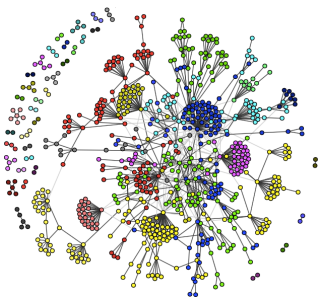
University of Geneva

Studying networks



Network as input

→ centralized algorithms...

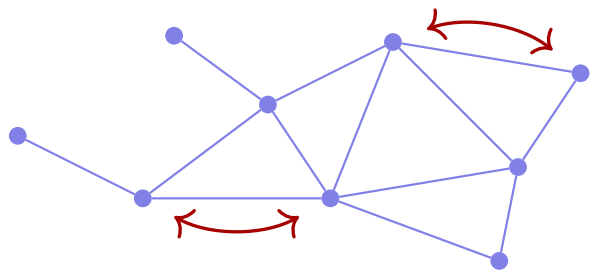


Network as environment

→ decentralized algorithms...
(a.k.a. distributed)

Distributed Algorithms

(Think globally, act locally)



Collaboration of distinct entities to perform a common task.

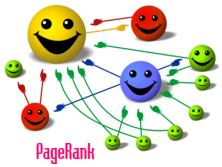
No centralization available, interactions among neighbors.

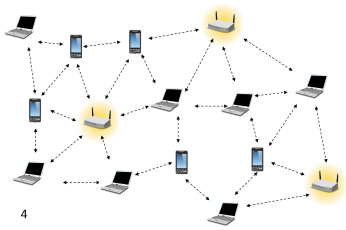
Theoretical aspects of collective intelligence.

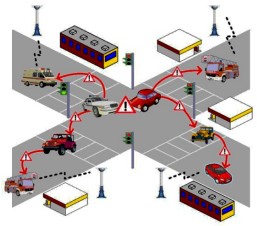
The world is distributed...

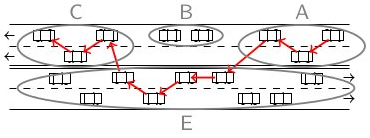
In technologies

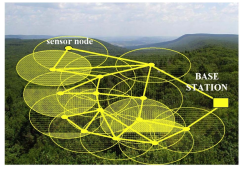


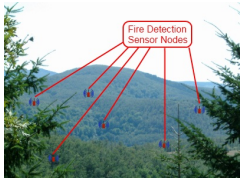


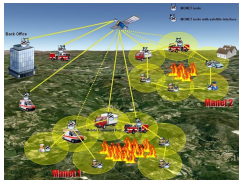


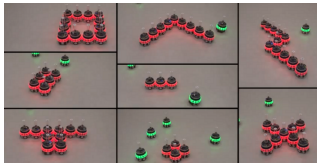
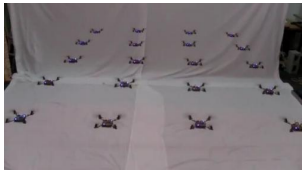










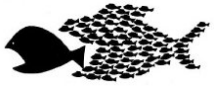
In nature

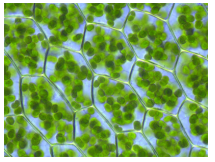
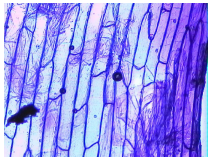






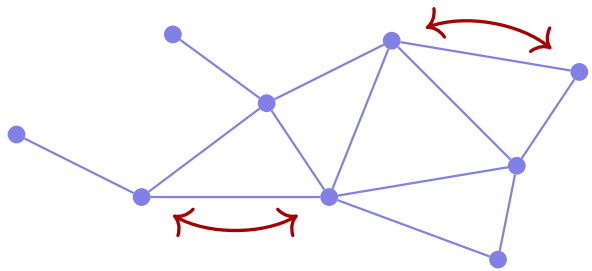





Distributed Algorithms

(Think globally, act locally)



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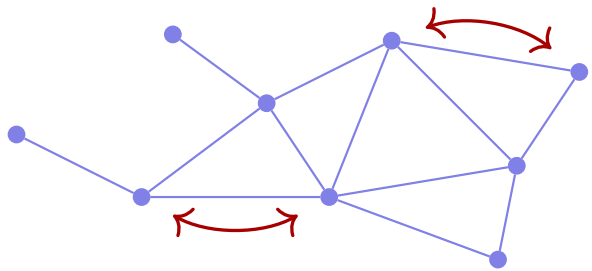

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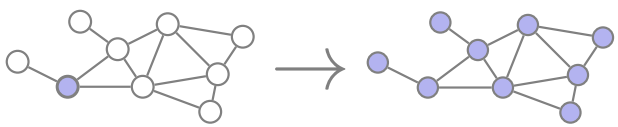
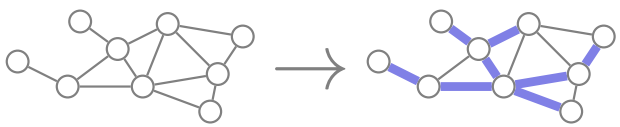
Examples of problems:

Broadcast



Spanning tree



Election

Counting

Consensus, naming, routing, exploration, coloring, dominating sets, ...

Two basic examples

Broadcasting algorithm:

```
onSelection():  // selected node
  informed ← true
onStart():  // all nodes
  if (informed)
    sendAll(new Message())
onMessage(message):
  if (¬ informed)
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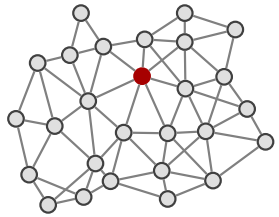

initially

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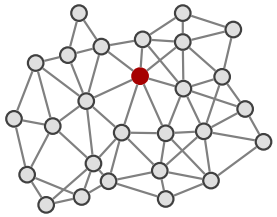
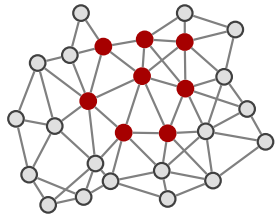


initially



a bit later

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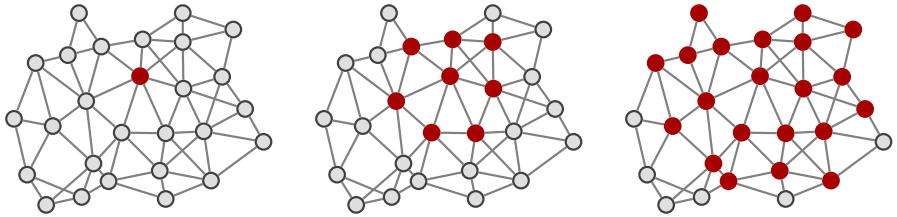

initially | a bit later | later

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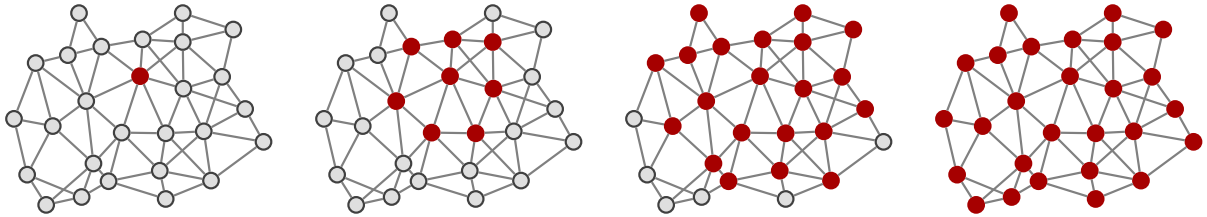

initially | a bit later | later | finally

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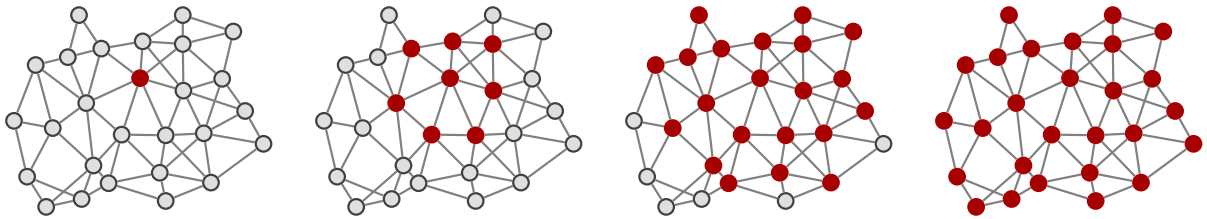

Spanning tree construction:

```
onSelection():  // selected node
  root ← true
onStart():  // all nodes
  if (root)
    sendAll(message)
onMessage(message):
  if (parent == null)
    parent ← message.sender
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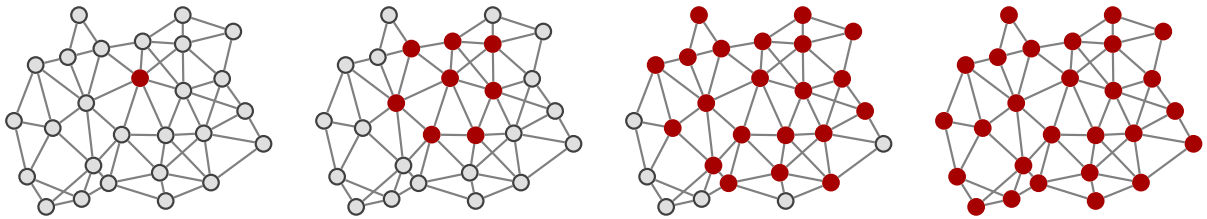

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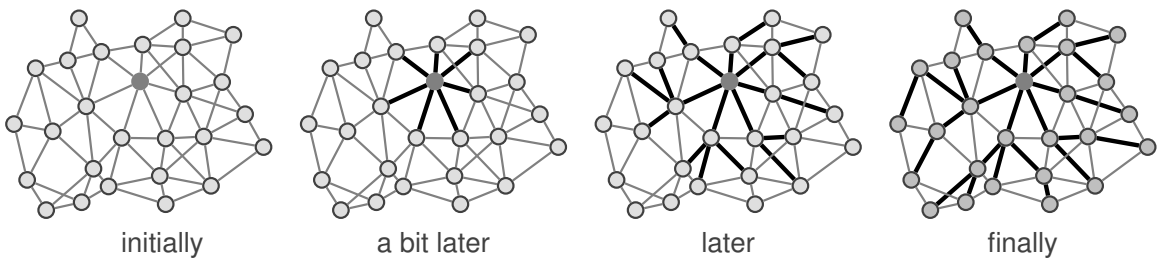

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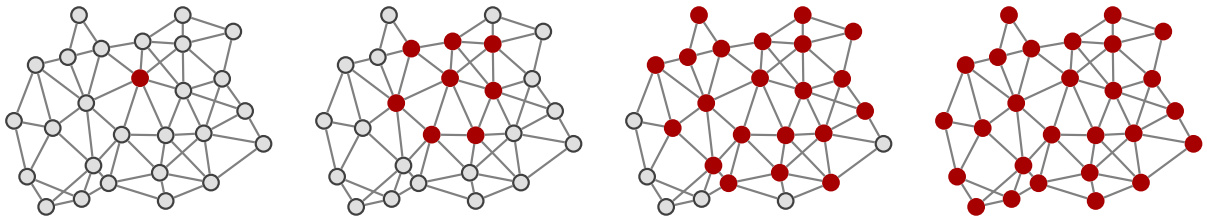

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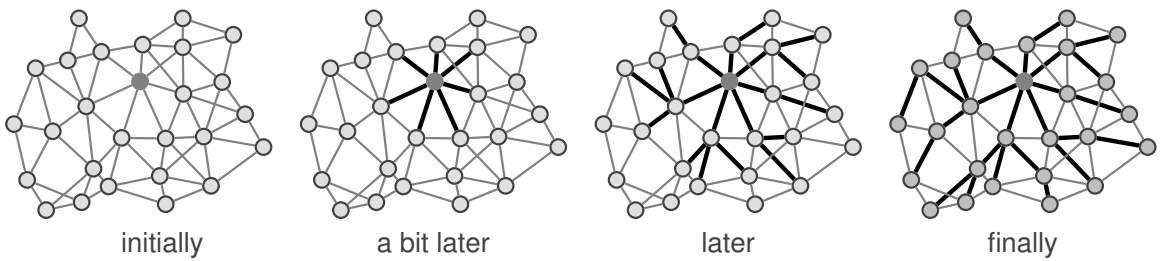
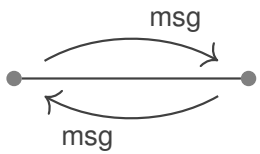


Everybody has the same algorithm, but initial states may differ (uniform versus non-uniform initialization).

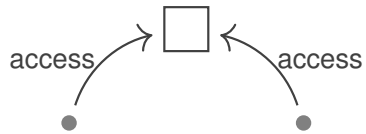
→ Various *modeling assumptions*

What assumptions?

Communication model:



Message passing



Shared memory

Other models : link registers, mobile agents, graph relabelings, (in nature) sounds, vision, smell, tactile, *etc.*

What assumptions?

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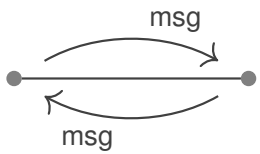

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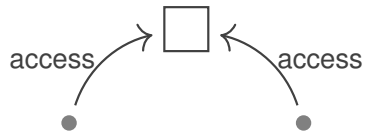

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→ Most frequent in networks: Message passing.

What assumptions?

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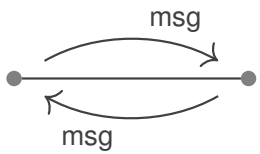

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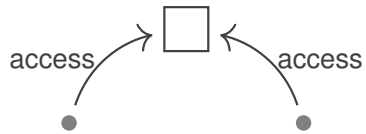

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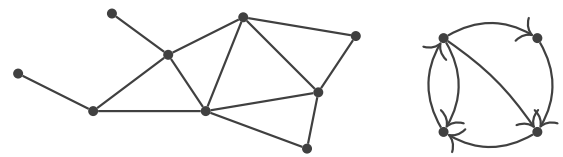
Modeling of the network:

→ Directed or undirected graph whose edges are communication links.



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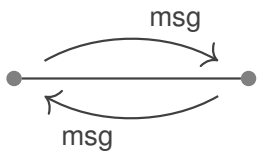

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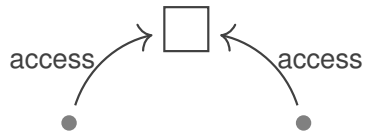

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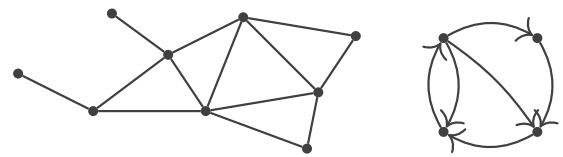

$\rightarrow$ Directed or undirected graph whose edges are communication links.

Other assumptions:

- Unique identifiers versus anonymous networks.

- Uniform versus non-uniform initialization (e.g. distinguished node, like in the previous slide).

- Synchronous versus asynchronous (are the rounds of communication synchronized?).

First algorithm in literature

Election in a **directed cycle**, with **synchronous communication**, **unique identifiers**, and **uniform** initialization. (Le Lann, Chang and Roberts, 1970's)

The node with the highest identifier gets elected.

onStart():

```
send(myID)
```

onMessage(otherID):

```
if (otherID > myID){
    send(otherID)
}else{
    if (otherID == myID){
        ELECTED ← true
    }
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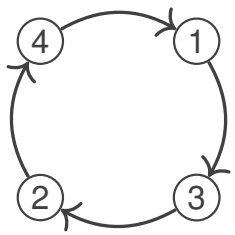

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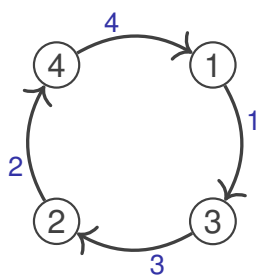

round 1

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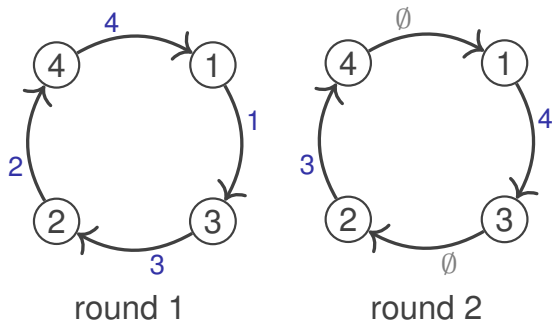

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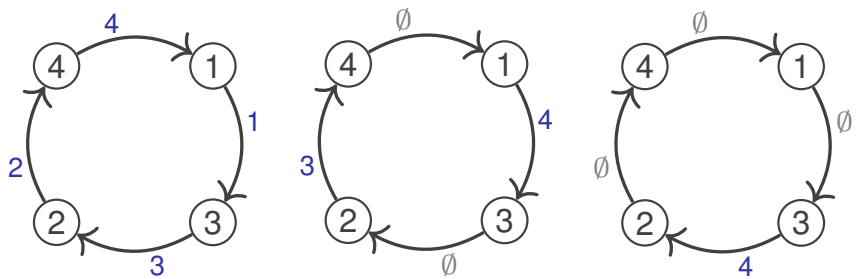

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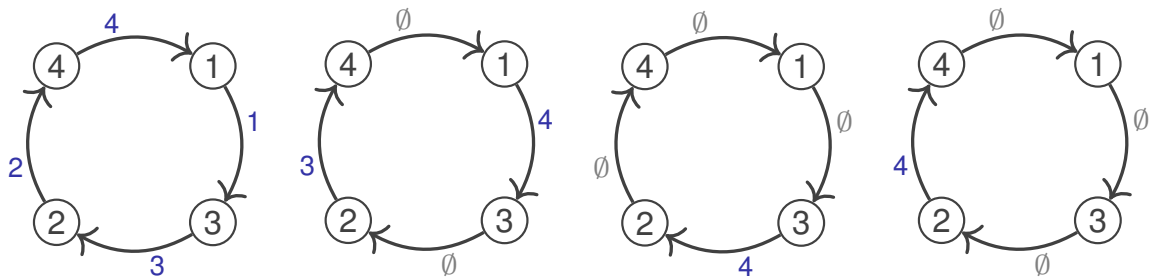

First algorithm in literature

Election in a **directed cycle**, with **synchronous communication**, **unique identifiers**, and **uniform** initialization. (Le Lann, Chang and Roberts, 1970's)

The node with the highest identifier gets elected.

onStart():

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onMessage(otherID):

```
if (otherID > myID){
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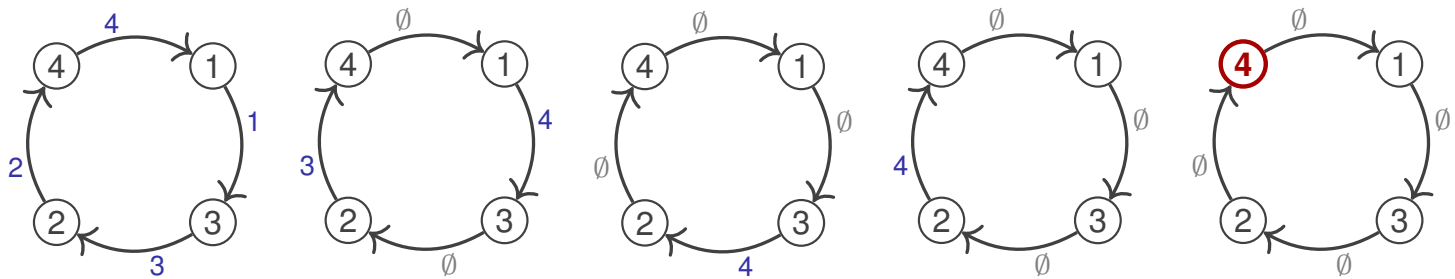

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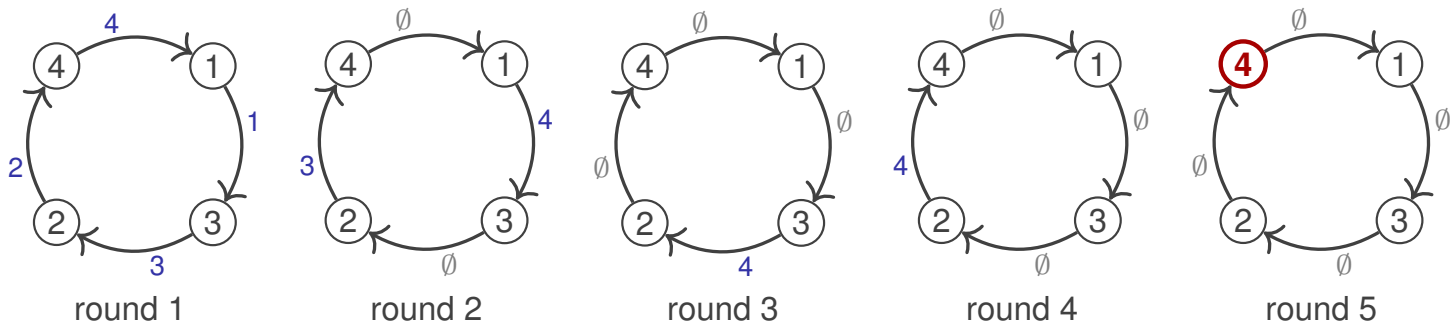

Worst-case time complexity?

Worst-case message complexity?

First algorithm in literature

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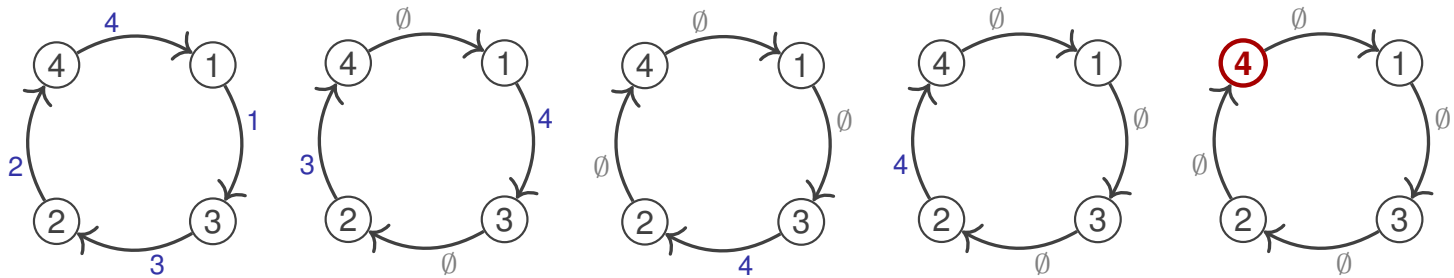

Worst-case time complexity? $\Theta(n)$

Worst-case message complexity?

First algorithm in literature

Election in a **directed cycle**, with **synchronous communication**, **unique identifiers**, and **uniform** initialization. (Le Lann, Chang and Roberts, 1970's)

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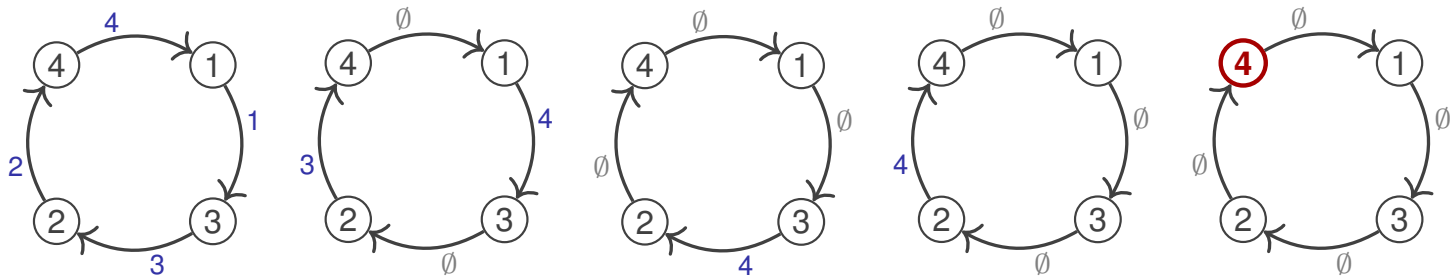
```

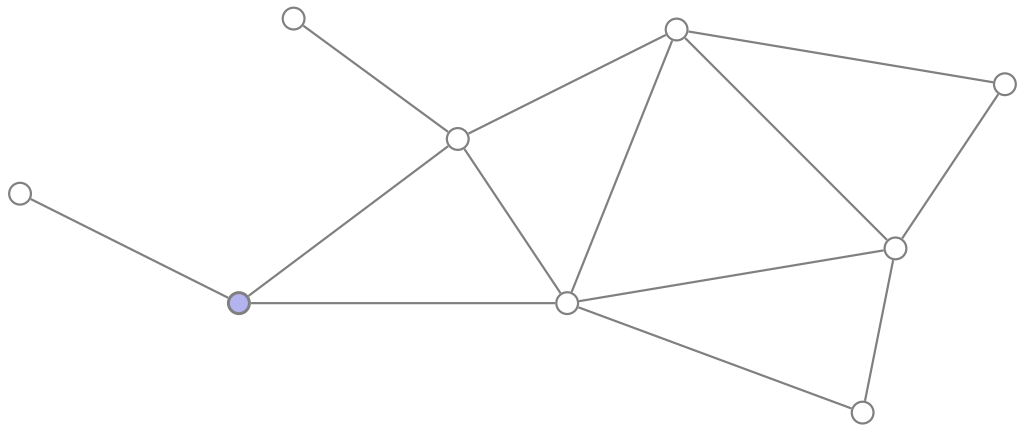


Worst-case time complexity? $\Theta(n)$

Worst-case message complexity? $\Theta(n^2)$

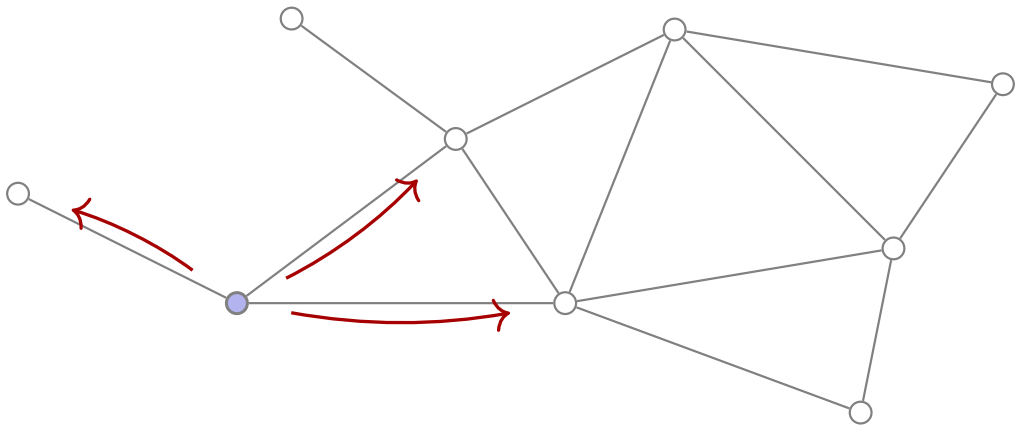
Complex example: Broadcasting | Spanning tree | Counting

Assumptions: synchronous communication / unique ids / distinguished node



Complex example: Broadcasting | Spanning tree | Counting

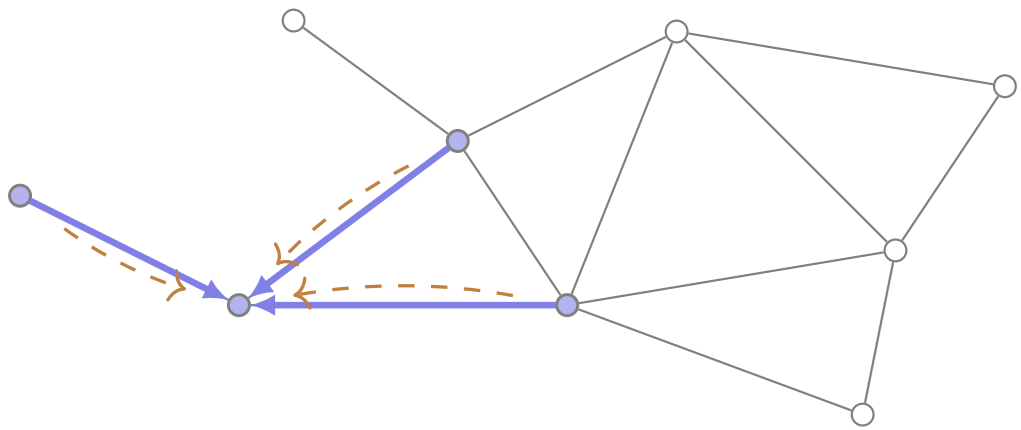
Assumptions: synchronous communication / unique ids / distinguished node



Round 1

Complex example: `Broadcasting` | `Spanning tree` | `Counting`

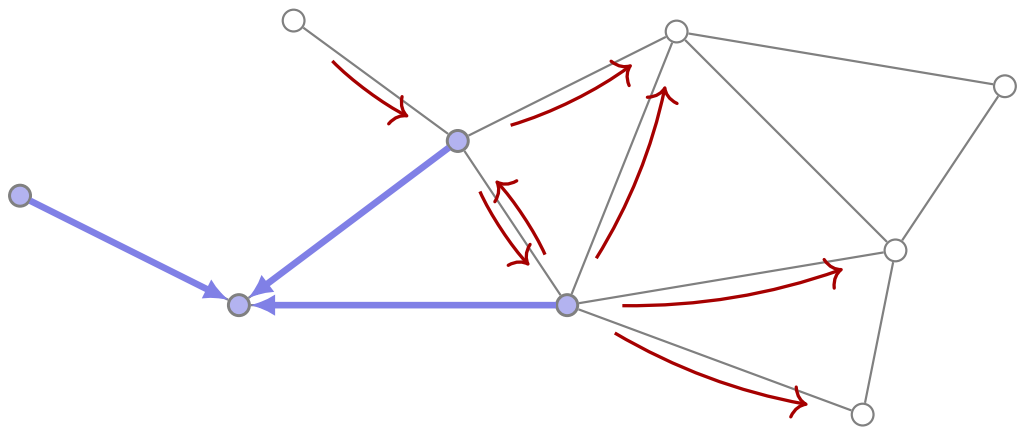
Assumptions: synchronous communication / unique ids / distinguished node



Round 2

Complex example: Broadcasting | Spanning tree | Counting

Assumptions: synchronous communication / unique ids / distinguished node



Round 2

Complex example: Broadcasting | Spanning tree | Counting

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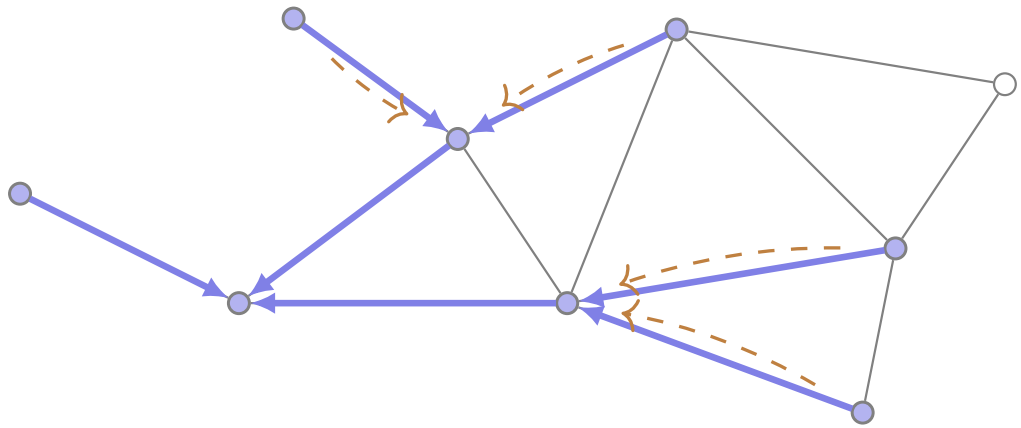
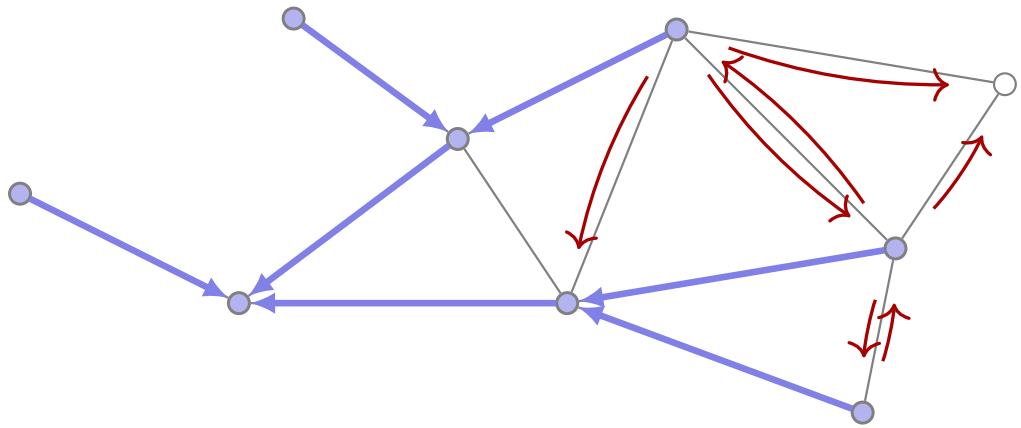


Round 3

Complex example: Broadcasting | Spanning tree | Counting

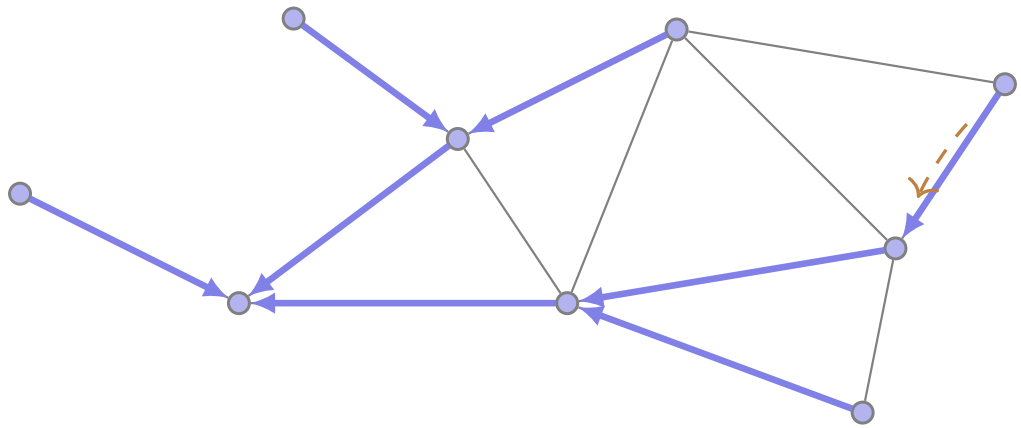
Assumptions: synchronous communication / unique ids / distinguished node



Round 3

Complex example: Broadcasting | Spanning tree | Counting

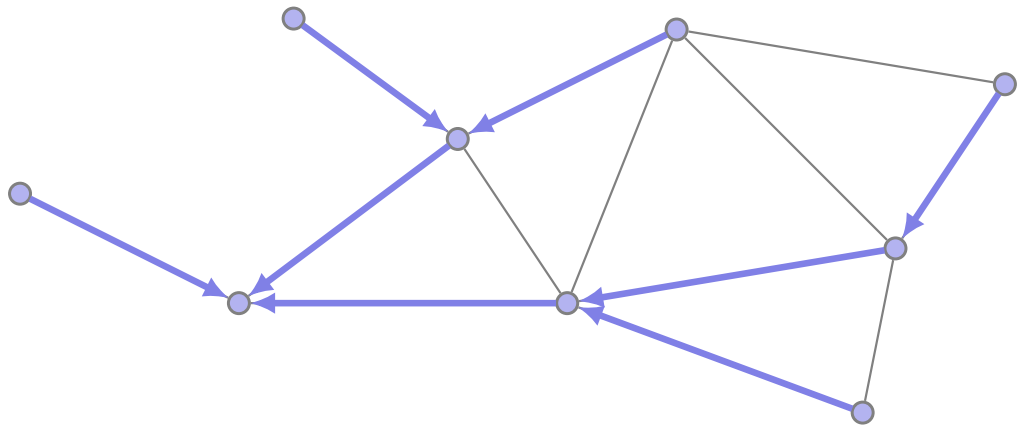
Assumptions: synchronous communication / unique ids / distinguished node



Round 4

Complex example: Broadcasting | Spanning tree | Counting

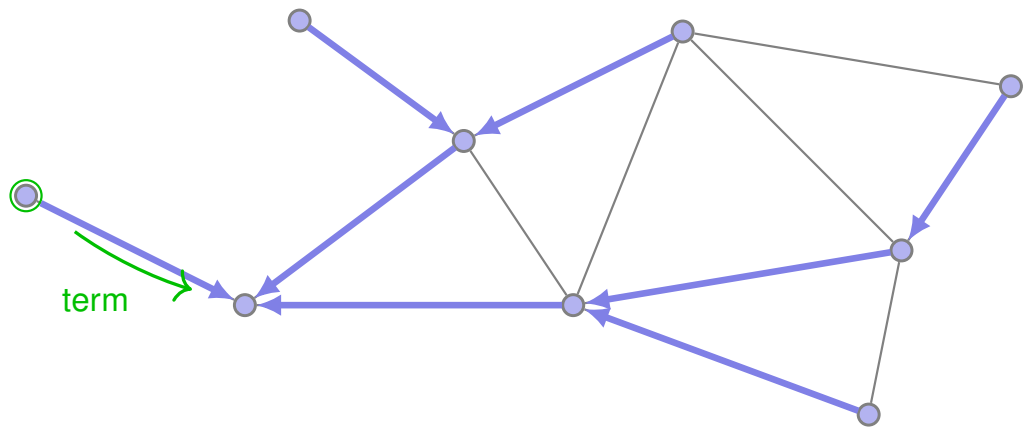
Assumptions: synchronous communication / unique ids / distinguished node



Round 4

Complex example: Broadcasting | Spanning tree | Counting

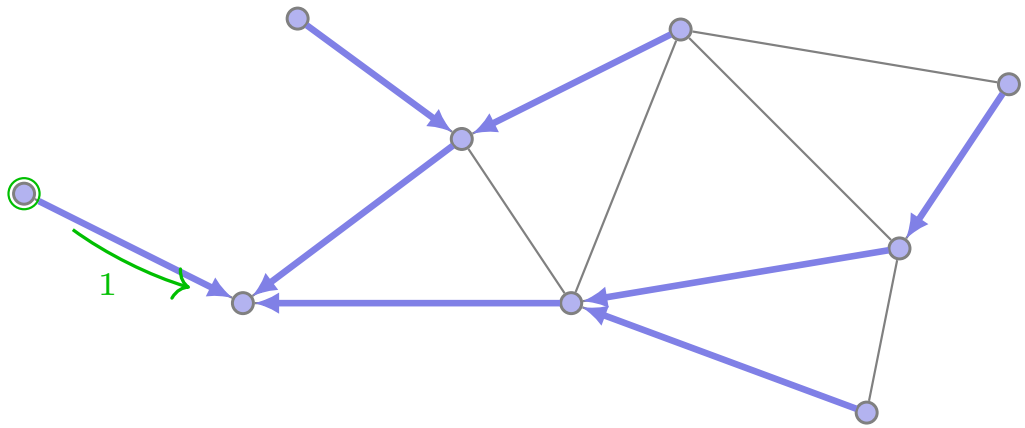
Assumptions: synchronous communication / unique ids / distinguished node



Round 4

Complex example: Broadcasting | Spanning tree | Counting

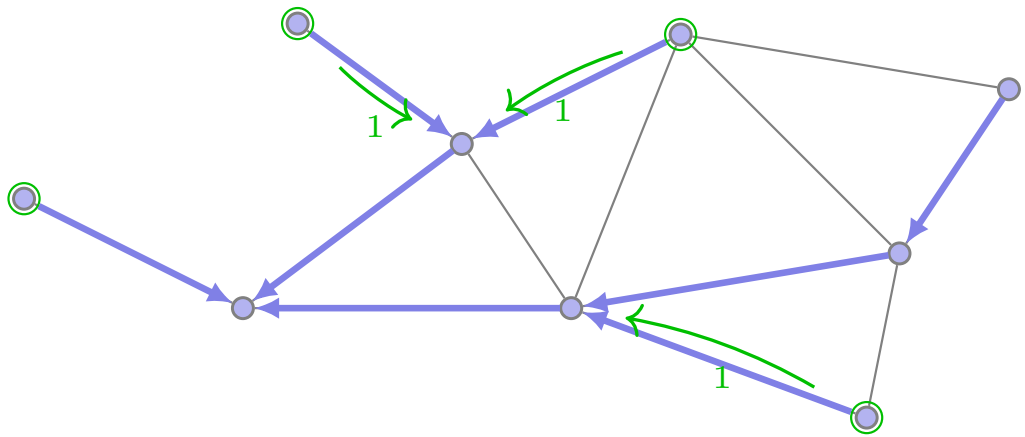
Assumptions: synchronous communication / unique ids / distinguished node



Round 4

Complex example: Broadcasting | Spanning tree | Counting

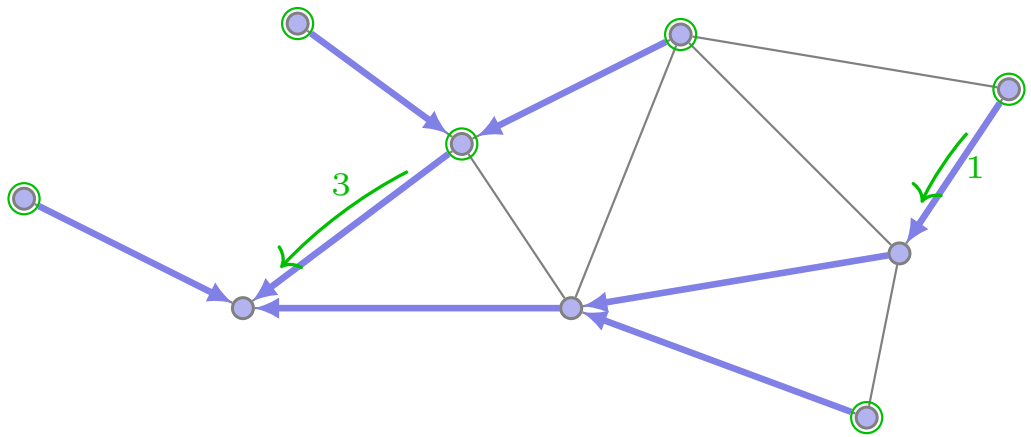
Assumptions: synchronous communication / unique ids / distinguished node



Round 5

Complex example: Broadcasting | Spanning tree | Counting

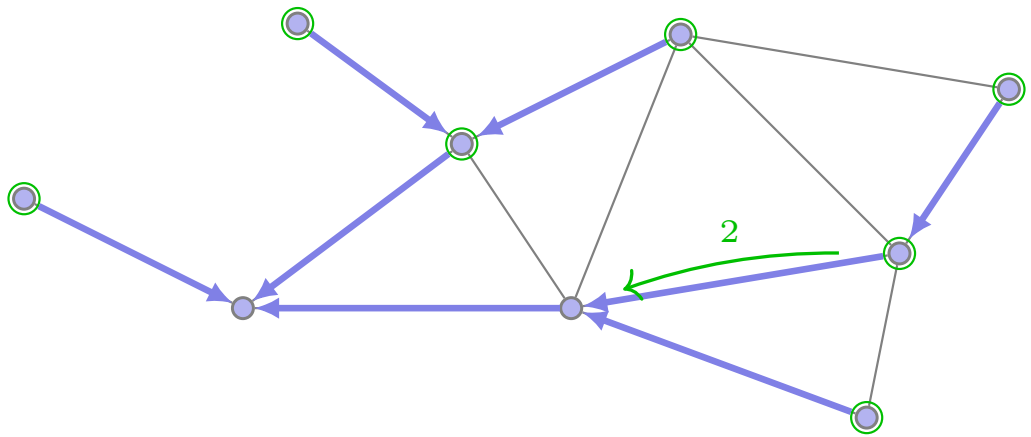
Assumptions: synchronous communication / unique ids / distinguished node



Round 6

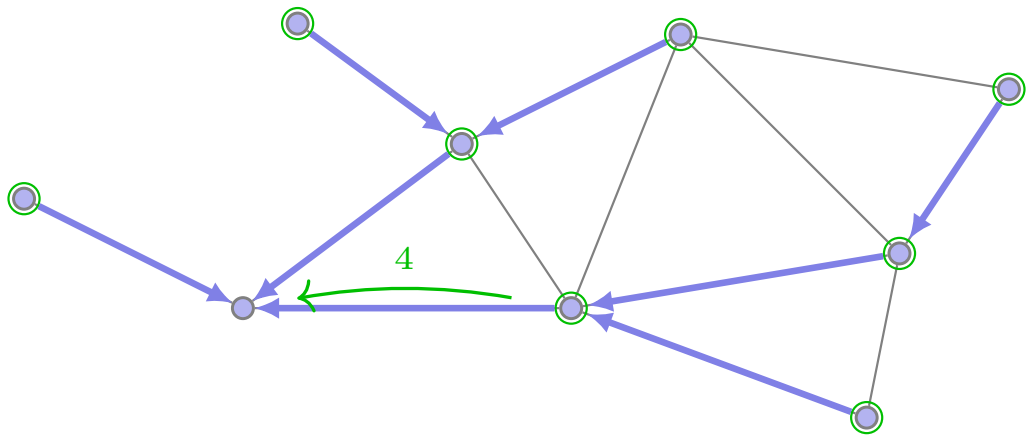
Complex example: Broadcasting | Spanning tree | Counting

Assumptions: synchronous communication / unique ids / distinguished node



Complex example: Broadcasting | Spanning tree | Counting

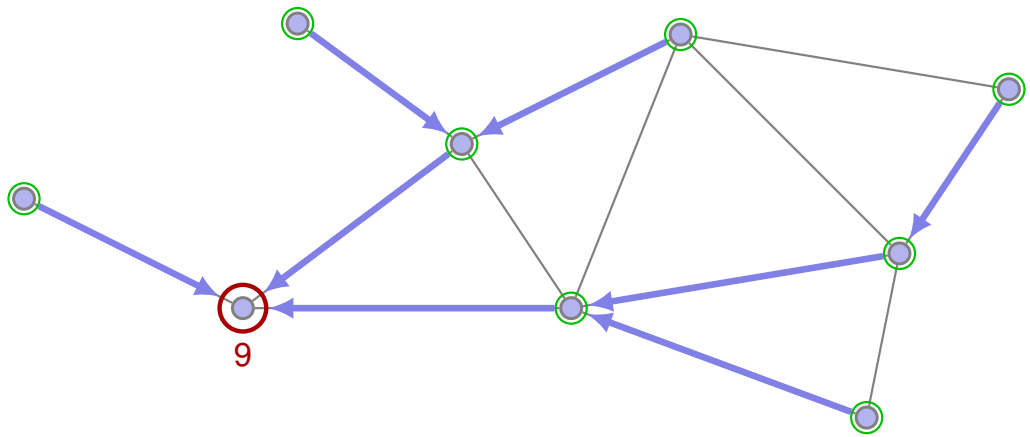
Assumptions: synchronous communication / unique ids / distinguished node



Round 8

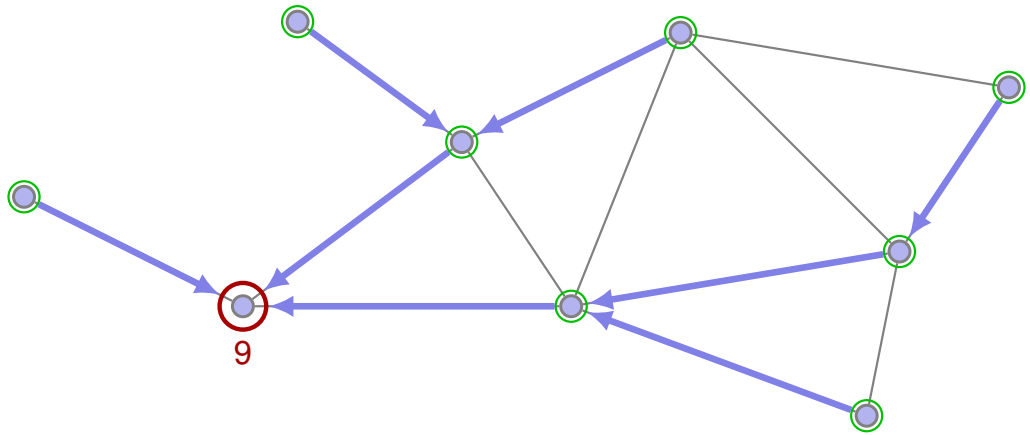
Complex example: Broadcasting | Spanning tree | Counting

Assumptions: synchronous communication / unique ids / distinguished node



Complex example: Broadcasting | Spanning tree | Counting

Assumptions: synchronous communication / unique ids / distinguished node



Time complexity: $O(\text{diameter}(G)) = O(n)$

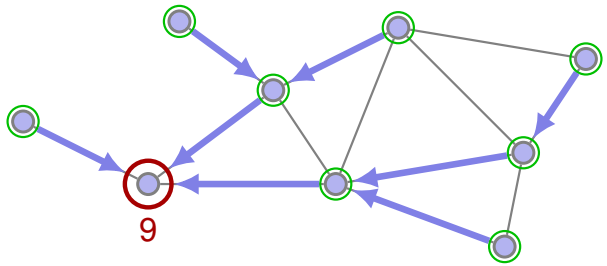
Message complexity: $O(m + n + n) = O(m)$

$n : \#nodes$

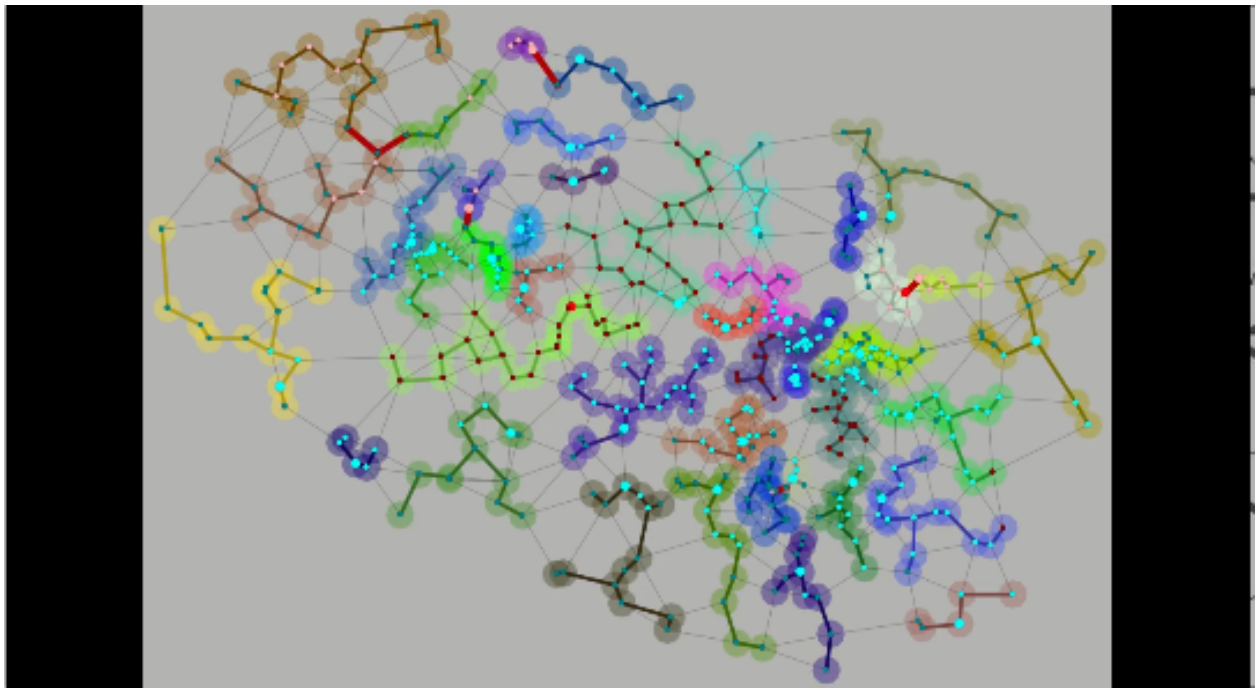
$m : \#edges$

Complex example: Broadcasting | Spanning tree | Counting

Assumptions: synchronous communication / unique ids / distinguished node

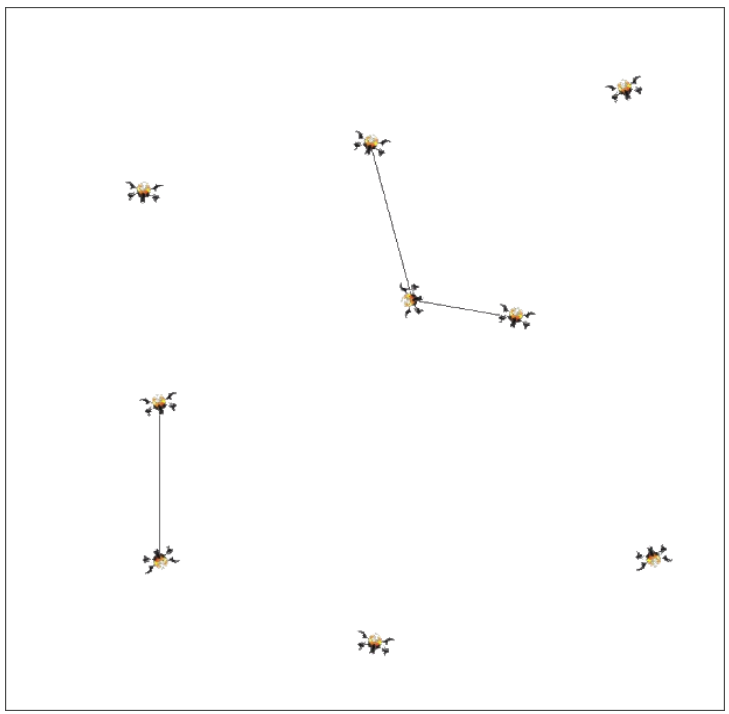


What if there are no distinguished nodes? (The GHS algorithm, 1983)



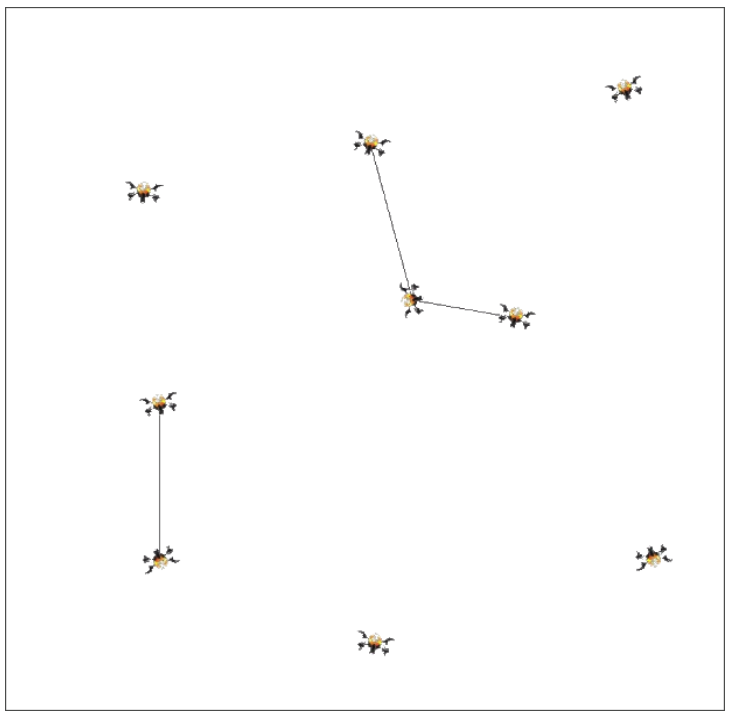
(Highly) dynamic networks?

Example of scenario

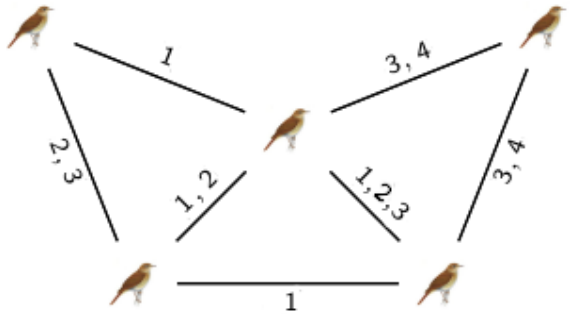


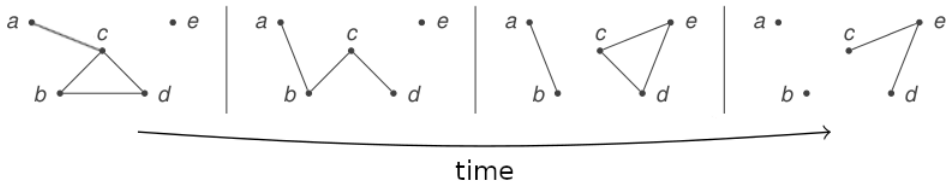
(Highly) dynamic networks?

Example of scenario



Modeling





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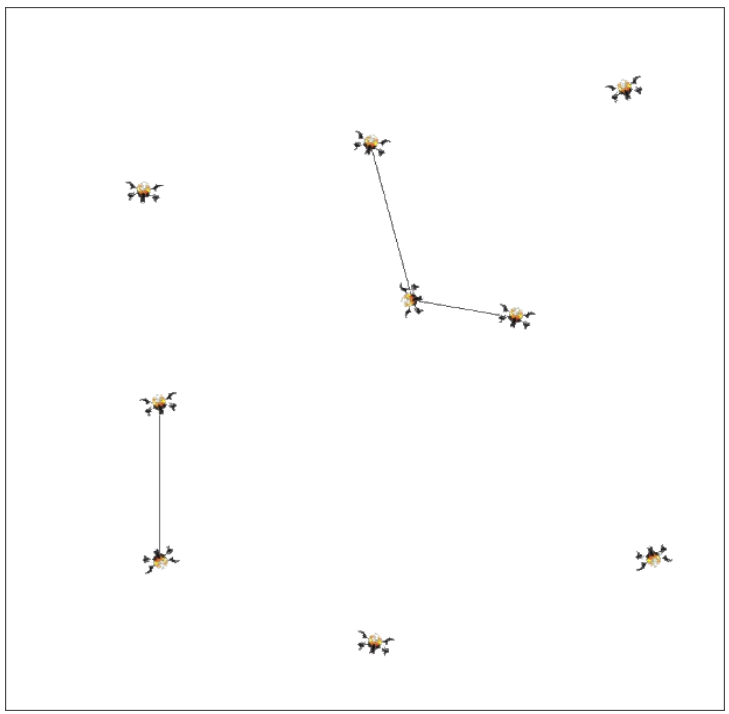

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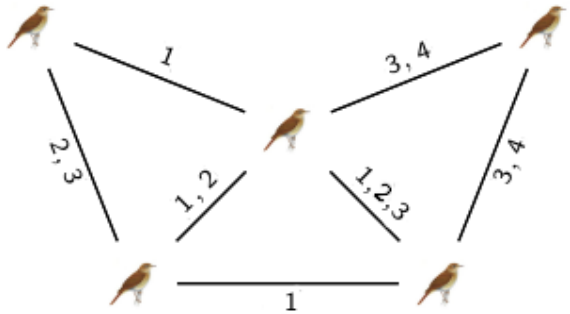

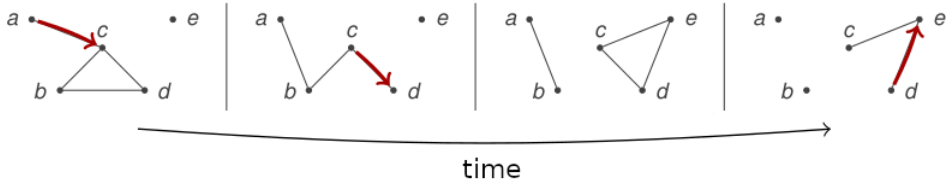

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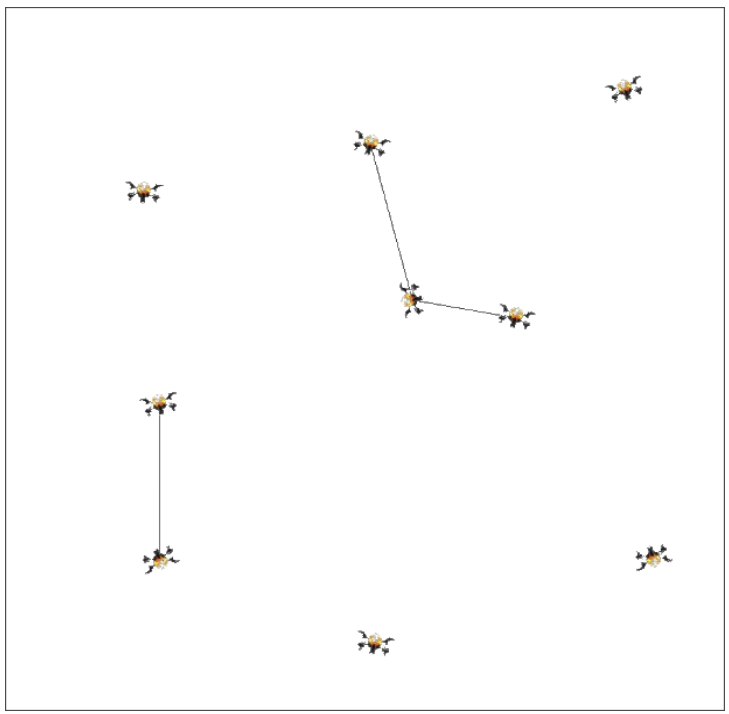

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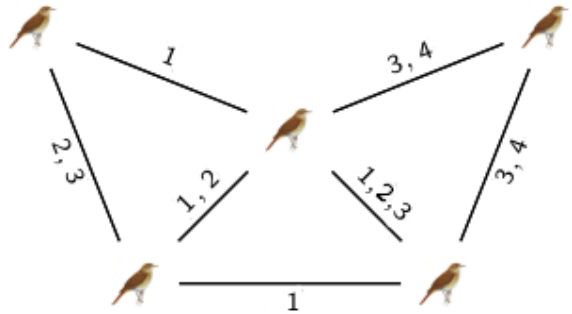
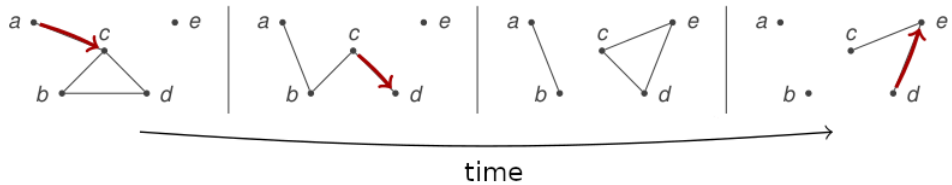


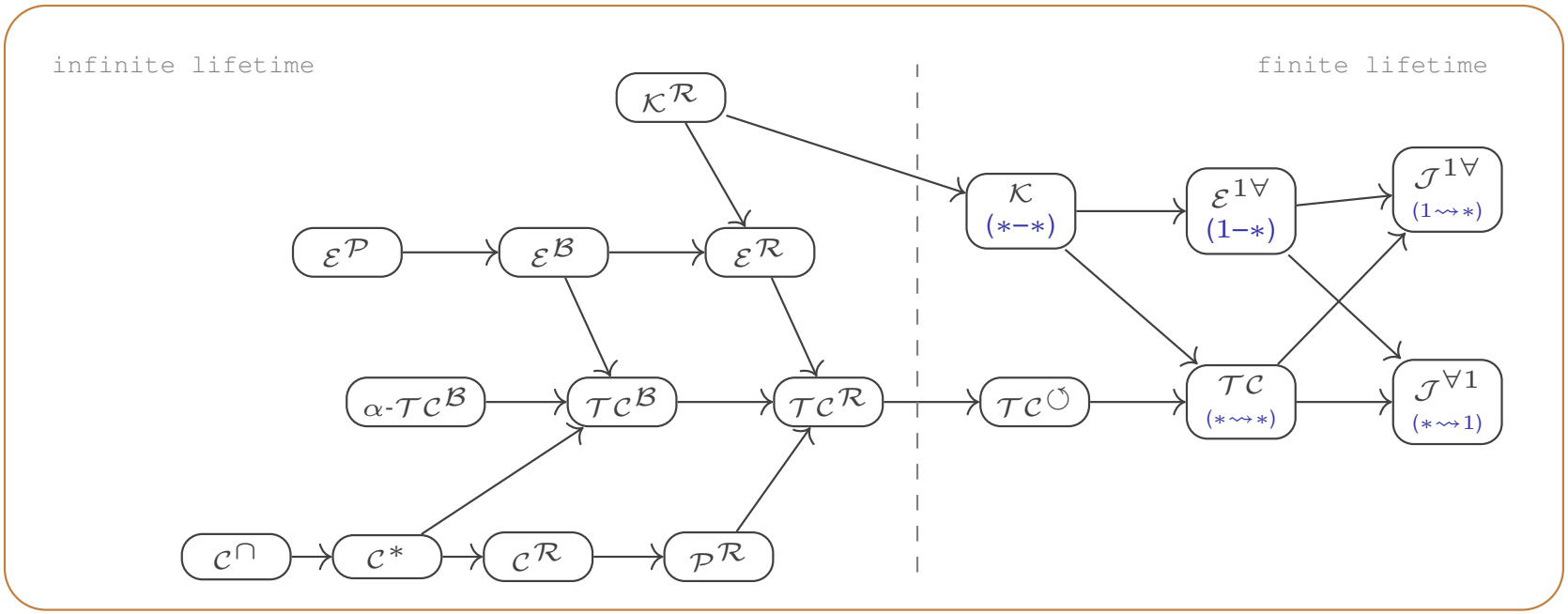


Properties:

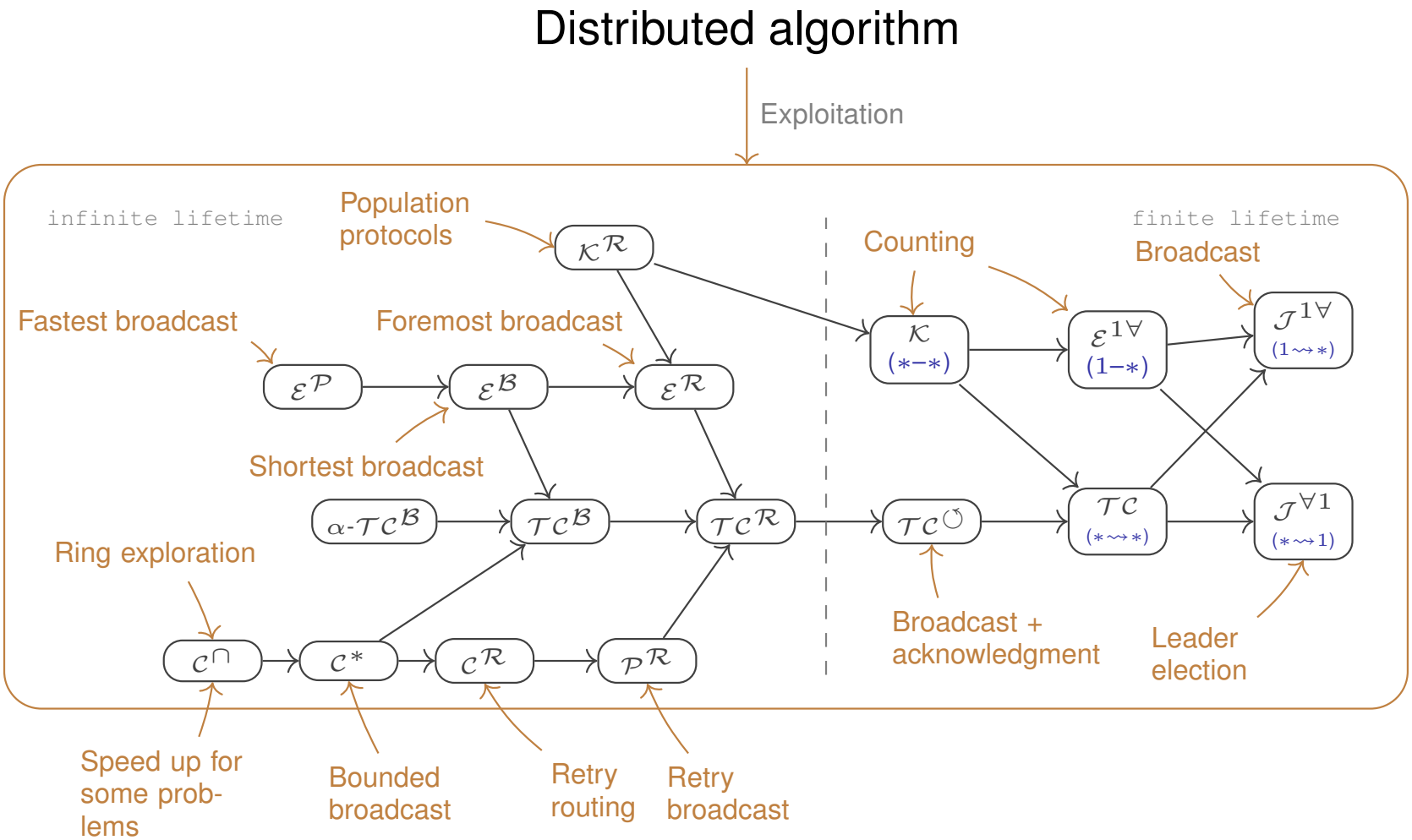
- Temporal connectivity? $\mathcal{TC}$
- Repeatedly? $\mathcal{TC}^{\mathcal{R}}$
- Recurrent links? $\mathcal{E}^{\mathcal{R}}$
- In bounded time? $\mathcal{E}^{\mathcal{B}}$
- ...

→ Classes of temporal graphs

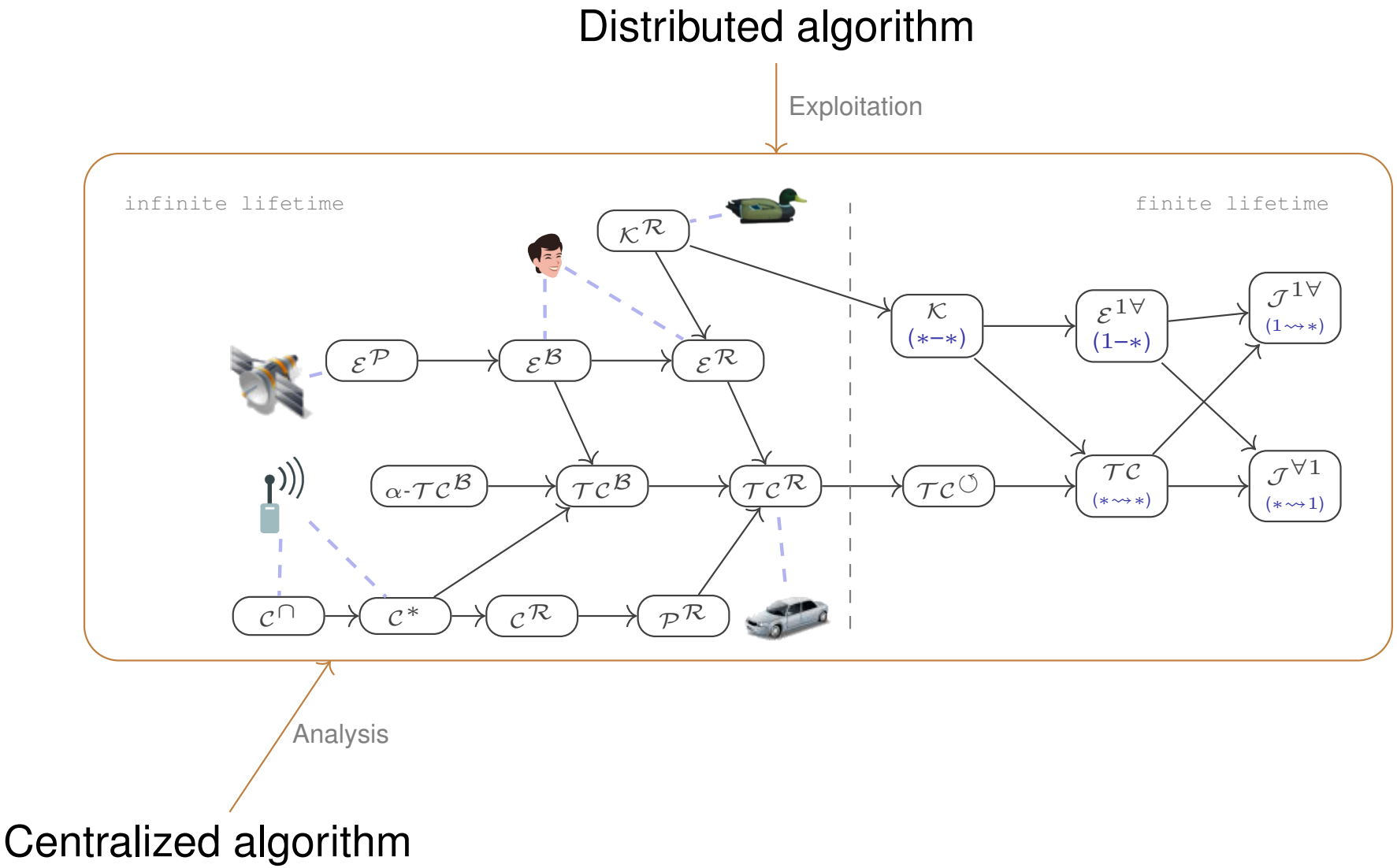
Some classes of temporal graphs



Some classes of temporal graphs



Some classes of temporal graphs



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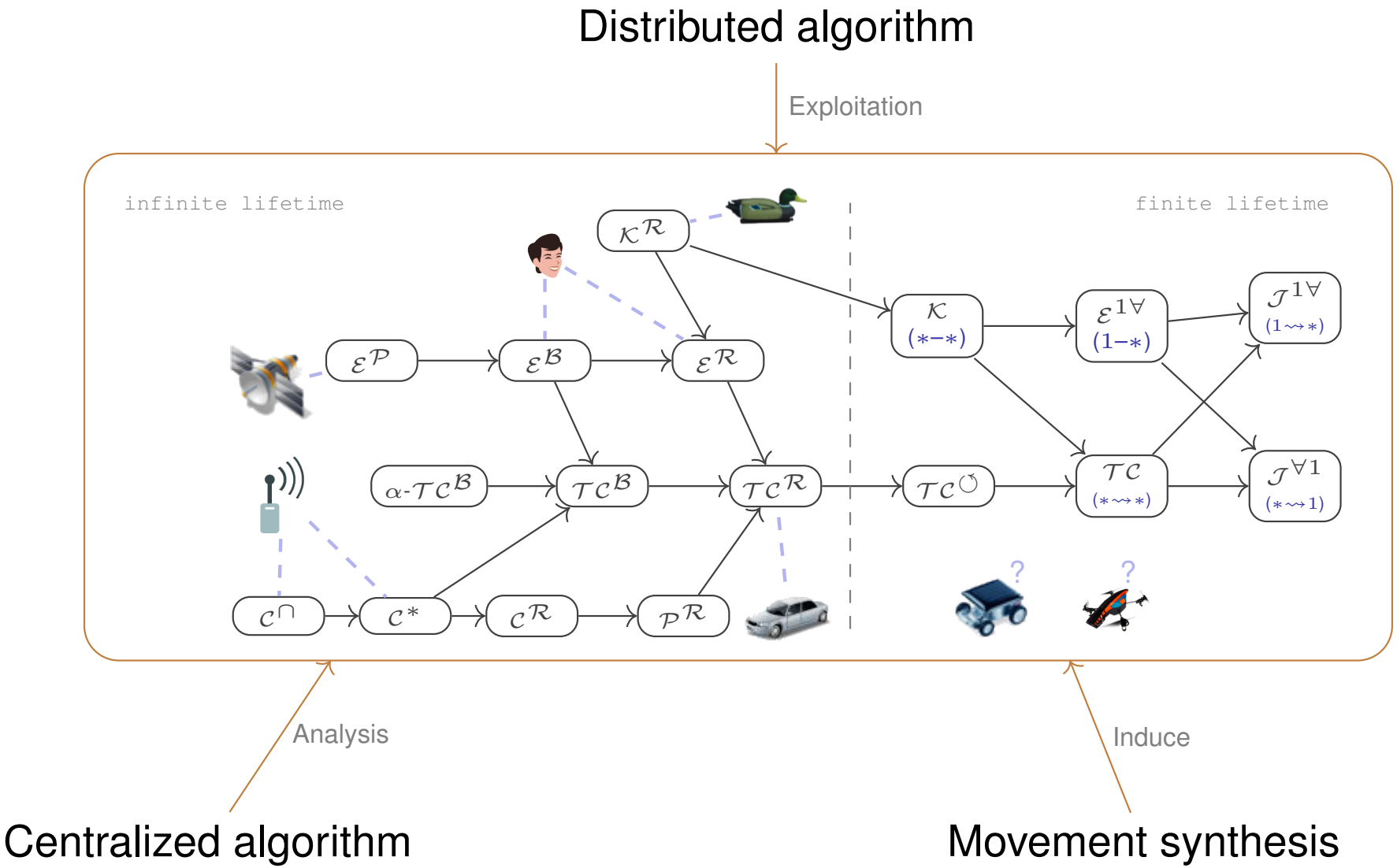
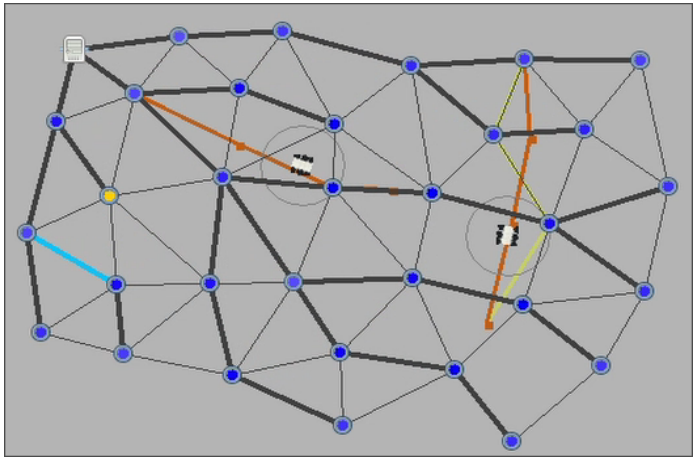
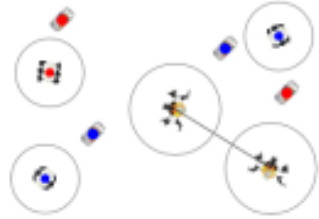


Tool for the exercise session

JBotSim: Collective intelligence / Network algorithms / Motion planning / Robotics

(h) Acceleration constraints

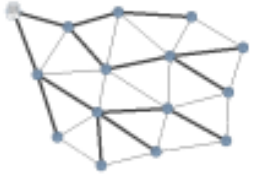




(i) Heterogeneous park cleaning

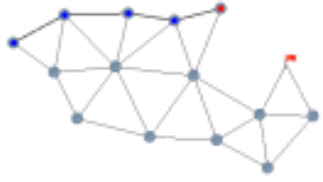
(k) Obstacle management

(o) Navigation (embedding)



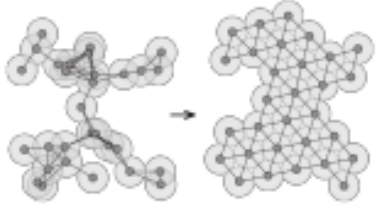
(a) Data aggregation

(b) Vehicular networks

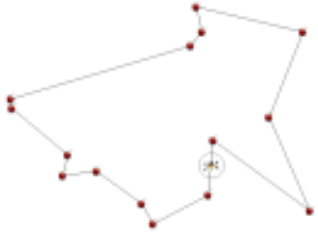


(c) Geograhical routing

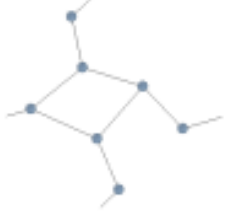
(g) Territory sharing



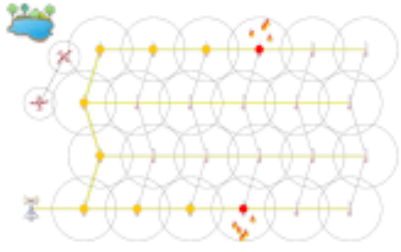
(d) Deployment by virtual forces



(e) Travelling Salesman Problem



(f) Toroidal space



(j) Fire-fighting aircrafts